PERFORMANCE EVALUATION OF DIRECTION-OF-ARRIVAL (DOA) ESTIMATION ALGORITHMS FOR MOBILE COMMUNICATION SYSTEMS

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Abstract – Spatial filtering for mobile communications has attracted a lot of attention over the last decade and is currently considered a very promising technique that will help future cellular networks achieve their ambitious goals. One way to accomplish this is via array signal processing with algorithms which estimate the Direction-Of-Arrival (DOA) of the received waves from the mobile users. This paper evaluates the performance of a number of DOA estimation algorithms. In all cases a linear antenna array at the base station is assumed to be operating in a typical cellular environment.

I. Introduction

Adaptive Antennas have attracted a lot of attention over the past few years as possible solutions to some of the main problems associated with current mobile systems [1]. For this purpose a number of research activities are being carried out in order to properly design, analyse, and implement adaptive antennas for mobile communications applications, *e.g.* TSUNAMI Project (Technology in Smart antenna for the UNiversal Advanced Mobile Infrastructure) [2]. Direction of arrival (DOA) estimation algorithms are used to improve the performance of an antenna by controlling the directivity of the antenna to reduce effects like interference, delay spread, and multipath fading. Several DOA estimation algorithms are available which are categorised by Krim and Viberg [3] into three categories: spectral estimation, parametric subspace-based estimation (PSBE), and deterministic parametric estimation (DPE). The most researched algorithm of the first category is the MUSIC (MUltiple SIgnal Classification) algorithm [4]. ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) [5] and all its variants such as, LS- (Least Square), TLS- (Total Least Square) [6], and Unitary-ESPRIT [7] belong to the PSBE techniques. The DPE techniques include: Maximum Entropy (ME) [8], Maximum Likelihood (ML) [9], Space-Alternating Generalised Expectation-maximisation (SAGE) [10], and Weighted Subspace Fitting (WSF) [11] methods.

In this paper some of the most researched DOA estimation algorithms, *i.e.* Root-MUSIC, standard ESPRIT and Unitary-ESPRIT, are tested in different propagation environments. The modelling of the different environments is done with the propagation model that was developed by Piechocki and Tsoulos in [12]. This model is a combination of the Geometrically Based Single Reflection (GBSR) and the Gaussian Wide Sense Stationary Uncorrelated Scattering (GWSSUS) models with temporal variations, and was specifically developed for studying adaptive antennas in wideband mobile communication systems.

The aim of this study is to provide a comprehensive comparison of the most widely discussed and researched DOA estimation algorithms (on the same basis). The results from such a comparison can then be used to indicate solutions for different levels of applications, *e.g.* for measurement systems with the capability to provide spatial information, for cellular base stations with the capability to improve range-capacity-service quality *etc.*, for user positioning systems, and many more.

II. Basic Channel Model

Consider a uniformly spaced, linear array consisting of N sensors on which plane waves from M ($M < N$) narrow-band sources impinge from directions $\theta_1, \ldots, \theta_M$. Taking the first element in the array as the phase reference and assuming that the signal sources are in the far-field, the complex vector received by the array can be expressed as [13]:

$$y(t) = \sum_{m=1}^{M} a(\theta_m) s_m(t) + w(t) \tag{1}$$

where $s(t)$ is the signal vector, $w(t)$ is the additive noise vector and $a(\theta)$ is referred to as the *array response* or *array steering vector* for the direction θ, *i.e.*:

$$\begin{aligned} a(\theta) &= [1 \quad e^{-j\phi} \quad \cdots \quad e^{-j(N-1)\phi}]^T, \\ \phi &= 2\pi(d/\lambda)\sin\theta \end{aligned} \tag{2}$$

where ^{T} is the transposition operator, d in the spacing between sensors and λ is the wavelength of the received signal.

Representing (1) in a more compact matrix form of size $N\times1$:

$$y(t) = As(t) + w(t) \tag{3}$$

where $s(t)$ is $M\times1$ vector and $A=[\,a(\theta_1), \ldots, a(\theta_M)]$ is the $N\times M$ matrix of steering vectors.

We assume that the $w(t)$ is modelled as temporally white and zero-mean complex Gaussian process. Now, the $N\times N$ spatial correlation matrix of the observed signal vector $y(t)$ can be defined as:

$$R = E[y(t)y^H(t)] \tag{4}$$

where $E[\cdot]$ and ^{H} denote expectation operator and conjugate transpose, respectively. R is sometimes also referred to as the *array covariance matrix*. From our assumptions this can also be represented as follows (recognising also that $s(t)$ and $w(t)$ are statistically independent):

$$R = ASA^H + \sigma_w^2 I \quad (5)$$

where S is the signals' covariance matrix ($S=E[s(t)s^H(t)]$), σ^2 is the variance of the noise and I denotes an $N\times N$ identity matrix.

The subspace methods utilise the special eigenstructure of R which is expressed in terms of its eigenvalues, λ_n, and their corresponding eigenvectors e_n ($n=1,2,\ldots,N$). We assume that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N$. The first M eigenvalues will correspond to the directional sources and their values are larger than σ^2, and the remaining ($N-M$) eigenvalues are equal to σ^2. The eigenvectors corresponding to the signal eigenvalues can be used to describe the *signal subspace* : $E_s = [e_1 \ldots e_M]$. E_n is the matrix containing the remaining N-M noise eigenvectors that describe the *noise subspace*, which is the orthogonal complement to the signal subspace: $E_n = [e_{M+1} \ldots e_N]$.

The estimation of the number of sources M is carried by using information theoretic criteria [14] *e.g.* Akaike's or the Minimum Description Length (MDL) criterion [15].

MUSIC

The MUSIC algorithm developed by Schmidt [4] tends to exploit the orthogonality between the signal subspace and noise subspace. This is done by searching for peaks in the MUSIC spectrum which are a function of the look direction, θ, given by:

$$P_{MU}(\theta) = \frac{1}{a^H(\theta)E_n E_n^H a(\theta)} \quad (6)$$

The peaks in the spectrum occur at the points where the steering vector is orthogonal to the noise subspace, *i.e.* the denominator in (6) goes to zero and therefore $P_{MU}(\theta)$ peaks. The accuracy of the signal arrival directions estimated by MUSIC depend on the accuracy of the correlation matrix R. This can be improved by using more snapshots of data and/or having high Signal-to-Noise Ratio (SNR) [16]. Also, the accuracy of MUSIC estimate degrades when the incident signals are coherent, which can be improved by using spatial smoothing preprocessing before the MUSIC algorithm [17].

Root-MUSIC is a modified version of the MUSIC algorithm in which the DOA's are determined from the roots of a polynomial formed from the noise subspace [18]. Unlike MUSIC which is applicable to general array configurations, Root-MUSIC is restricted to uniform linear arrays. If we define polynomials using the noise eigenvectors [19]:

$$D_k(z) = \sum_{n=1}^{N} e_{nk} z^{-(n-1)}, \qquad k = M+1, \cdots, N \quad (7)$$

where e_{nk} are components of E_n. The above polynomials have roots at $z = e^{j2\pi(d/\lambda)\sin\theta_i}, i = 1, \cdots, M$. Now define the polynomial:

$$Q(z) = \sum_{k=M+1}^{N} D_k(z) D_k^*(1/z^*) \quad (8)$$

It can be seen that $Q(z)$ will have the same roots as $D_k(z)$ and there will be M double roots lying on the unit circle in the z-plane. These roots will correspond to the actual incident signals and the other roots which do not lie on the unit circle will not correspond to the signals and they are called spurious roots [8]. It has been reported in many studies that Root-MUSIC shows better performance than MUSIC especially in environments where the signals are located closer and/or they have low SNR [16], [19] and [20].

ESPRIT

ESPRIT [5] is a computationally efficient and robust method for estimating DOA which was developed in order to overcome the disadvantages of MUSIC. Other versions of ESPRIT have been developed to improve the technique, *e.g.* Least Squares (LS-ESPRIT), Total Least Squares (TLS-ESPRIT) [6], Unitary-ESPRIT [7]. Unitary-ESPRIT further reduces the computational complexity of the standard ESPRIT algorithm by using real-valued computations from start to finish. It not only estimates the DOA but it can be used to estimate the number of sources present. It also it incorporates forward-backward averaging which overcomes the problem of coherent signal sources. In this paper, the standard version of ESPRIT and Unitary-ESPRIT are tested.

III. Test Scenarios

The propagation scenarios used in testing the DOA estimation algorithms are divided into two main categories: *Macro-* and *Micro-cells*. This categorisation is based on the size of the cells and the height of the base station antenna. Macrocells are further divided, based on the environmental characteristics, into four sub-categories: *urban, bad urban, sub-urban,* and *rural.* Results for all these environments were produced according to the parameters described in [12] and the values suggested therein. The different DOA estimation algorithms were tested for the uplink of a W-CDMA (wideband-code division multiple access) signal transmitted by a single static mobile station (MS) and received at the base station (BS) by an 8-elements linear antenna array. The W-CDMA signal was generated following the ETSI UMTS Terrestrial Radio Access (UTRA) RTT proposal to ITU-R [21], as described in [22]. Table 1 summarises the values of some of the parameters employed in the test scenarios, (for more information refer to [12] and [22]).

Parameter	**Value**
Frequency (MHz)	1800
No. of Antenna Elements (N)	8
Inter-Element Spacing (d)	$\lambda/2$
Azimuth angle of MS (Degrees)	20
W-CDMA chip rate (k chips/sec)	4096
W-CDMA data rate (kbps)	16
SNR range (dB)	-5 to 20
Power Window (dB)	1, 5, 15

Table 1: Parameters used in the Tests

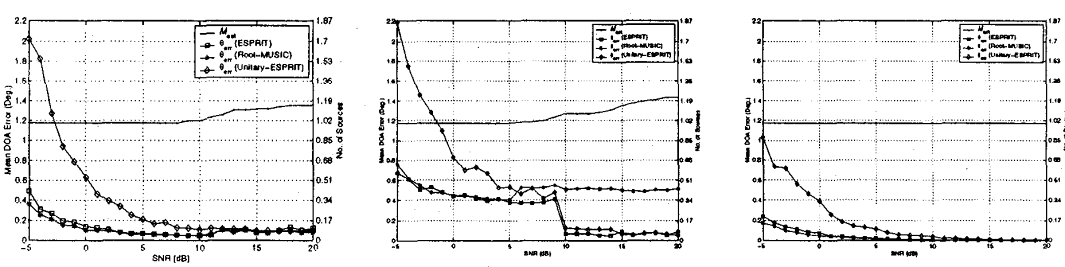
The comparison criterion is the mean DOA error of each DOA estimation algorithm when applied to the different simulated channel types with different values of power window for the impulse responses of the radio channel.

IV. Simulation Results

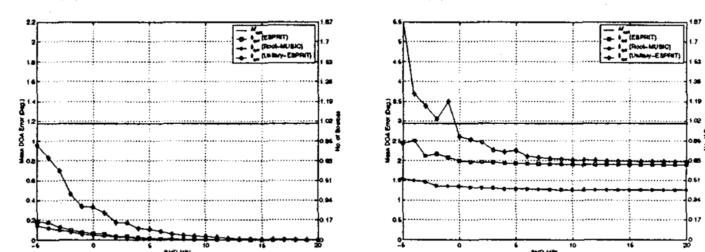
The channel impulse responses (CIR) for 100 snapshots were produced by the propagation model, for different channel types. The CIRs were then filtered with different power windows, applying the DOA estimation algorithms on the filtered CIRs for a range of SNR values and in order to estimate the number of sources present the MDL criterion was employed [15]. Figure 1 shows the mean (over 100 snapshots) of the DOA error ($\theta_{err} = \left|\theta - \tilde{\theta}\right|$) at each SNR level, for the different algorithms, for the different environments, with a power window for the CIRs of 1dB. The figure also includes the mean estimate of M found by the MDL algorithm for the same SNR range. It is evident from the figure that as the SNR level increases the DOA estimate error decreases with a different rate for the different environments, with the highest values for the Microcellular Urban scenario (Figure 1e). It can be seen that the tested DOA estimation algorithms have the smallest error in the Macrocell Sub-Urban and Rural scenarios, Figure 1c and d, respectively. This is due to that in these environments the BS receives small number of multipaths with relatively small angular spread. The Macrocell Urban and Bad Urban scenarios, Figure 1a and b, respectively, have a slightly higher level of DOA estimation error than the Macrocell Sub-Urban and Rural scenarios. This is due to the increase in the mean number of received multipaths that arrive at the BS with a higher angular spread. The DOA estimation algorithms produced the highest error in the Microcellular Urban environment, Figure 1e. This is due to that in this environment most of the time the BS receives at least two multipath with high angular spread. From the figures it can be seen that Root-MUSIC produced the best results for all environments except the Bad Urban case. Also the figures show that the DOA estimation error of Unitary-ESPRIT is the most sensitive to the change in the SNR levels as it produced high DOA error levels for low values of SNR that decrease by the increase in SNR.

When using a power window of 5dB, as Figure 2 shows, the performance of the DOA estimation methods degrades except from the Sub-Urban and Rural scenarios (Figure 2c and d, respectively). For the Macrocell Urban scenario (Figure 2a) the DOA estimate error reaches its lowest point at SNR≈7dB, for all the DOA estimation algorithms, and then increases slightly at the higher SNR levels. This is due to the effect of the increase in the number of sources estimated by the MDL algorithm. The effect of the additional cluster in the Bad Urban scenario (Figure 2b) is clear on the performance of the tested DOA estimation algorithms and on the Root-MUSIC in particular. The performance of the DOA estimation algorithms still exhibit the best performance in the Macrocell Sub-Urban and Rural scenarios, Figure 2c and e, respectively. Figure 2e shows that the DOA estimation error increased in the Microcell Urban scenario compared to the 1dB power window case, which shows that more multipaths with higher angular spread are present in the scenario compared to the other scenarios or to the same scenario with smaller power window.

Figure 3 shows results when a 15dB power window was used. From the figure it can be seen that for most of the DOA estimation algorithms the DOA estimate error is higher in all the propagation scenarios compared to the cases when using smaller power windows. This is mostly evident in the Macrocellular Bad Urban and Microcellular Urban scenarios, Figure 3b and e, respectively. This is due to the fact that when larger power windows are used more multipaths from different directions other than the required mobile direction might be also present. This will cause the MDL algorithm to estimate more sources in the environment. Since the DOA estimation algorithms use the estimate of M from the MDL algorithm, some of the distant scatterers will be considered as the actual required mobile. We have tried

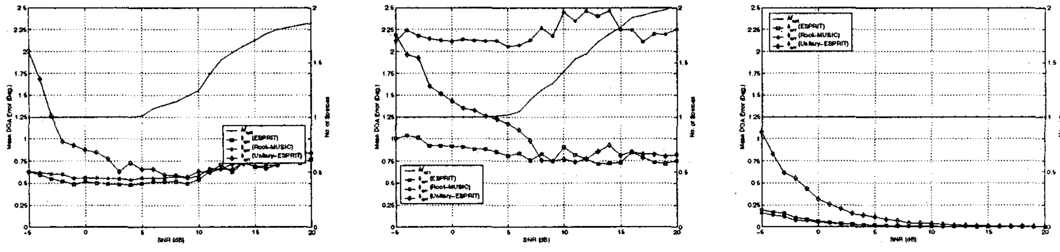


(a) Macrocell Urban (b) Macrocell Bad Urban (c) Macrocell Sub-Urban

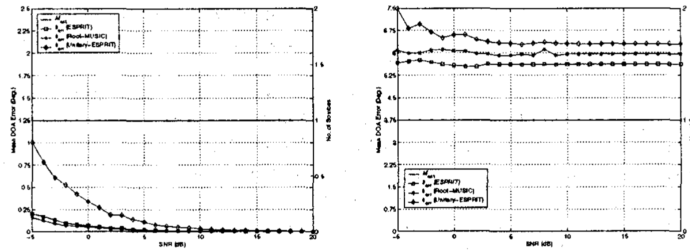


(d) Macrocell Rural (e) Microcell Urban

Figure 1: Mean DOA Error of the DOA estimation algorithms for the different propagation environments with 1dB Power Window.



(a) Macrocell Urban (b) Macrocell Bad Urban (c) Macrocell Sub-Urban



(d) Macrocell Rural (e) Microcell Urban

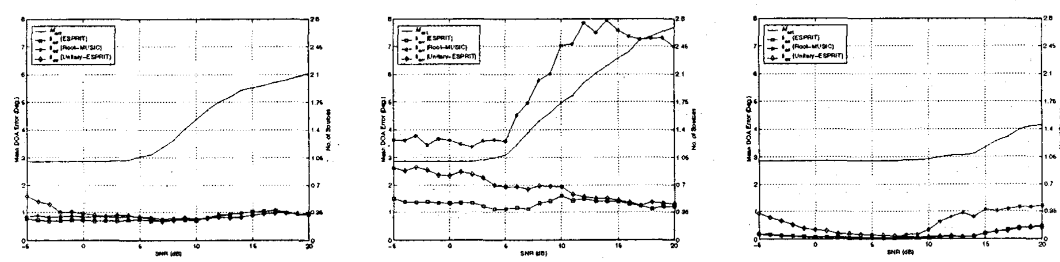
Figure 2: Mean DOA Error of the DOA estimation algorithms for the different propagation environments with 5dB Power Window.

to solve this problem by using the estimate of number of sources (M) calculated by the Unitary-ESPRIT from the most dominant eigenvectors [7] and tested it on the worst case scenarios with worst case power window. Figure 4 shows the results of the DOA estimate error in the Macrocell Urban, Macrocell Bad Urban and Microcell Urban scenarios when using the estimate of M calculated by the Unitary-ESPRIT algorithm with a power window of 15dB. The figure shows how the performance of the DOA estimation algorithms improved by using the better estimate of the number of sources. Figure 4b also shows that Root-MUSIC is more susceptible to the multipath cluster from the remote scatterers present in the Bad Urban scenario.

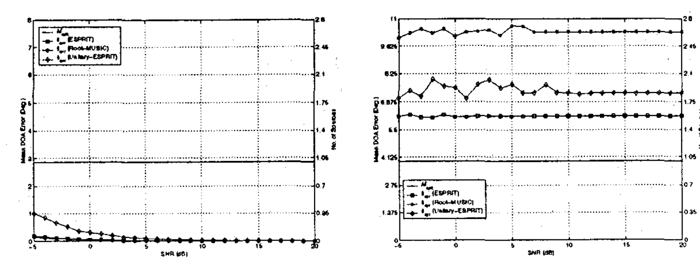
Figures 1 to 3 showed how the DOA estimation error increases when the power window size for the CIRs increases, especially in scenarios with more than one clusters of rays, *e.g.* Macrocell Bad Urban and Microcell Urban. Figure 4 showed how the performance of the DOA estimation techniques could be improved by employing more accurate estimate of the number of sources (M) present in the environment.

V. Conclusion

In this paper the performance of Root-MUSIC, standard version of ESPRIT and Unitary-ESPRIT have been tested in terms of the mean DOA error. This is carried out in five different scenarios: Macrocell Urban,

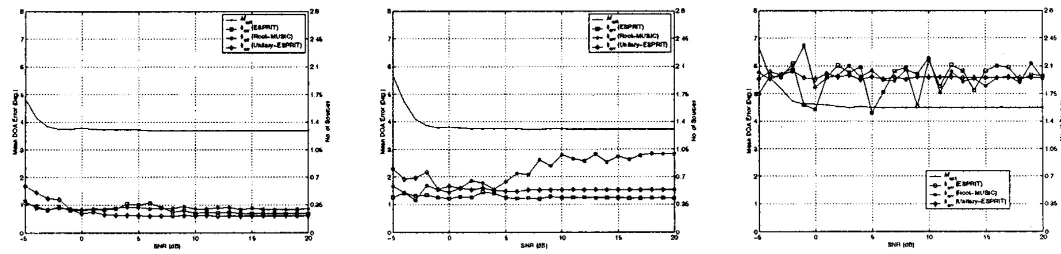


(a) Macrocell Urban (b) Macrocell Bad Urban (c) Macrocell Sub-Urban



(d) Macrocell Rural (e) Microcell Urban

Figure 3: Mean DOA Error of the DOA estimation algorithms for the different propagation environments with 15dB Power Window.



(a) Macrocell Urban (b) Macrocell Bad Urban (c) Microcell Urban

Figure 4: Mean DOA Error of the DOA estimation algorithms using estimate of M from Unitary-ESPRIT with a 15dB Power Window.

Macrocell Bad Urban, Macrocell Sub-Urban, Macrocell Rural and Microcell Urban. The algorithms have been applied at the BS to find the DOA of a W-CDMA signal transmitted by a single static MS. The received complex impulse responses (CIR) have been filtered using power window sizes of 1, 5 and 15dB for an SNR range from –5dB to 20dB. The results showed that Root-MUSIC outperformed the other algorithms in the Macrocell Sub-Urban and Rural scenarios, and for the Macrocell Urban scenario when the power window was small. However, Root-MUSIC produced high DOA estimation errors in scenarios with more than one clusters of rays, *e.g.* Macrocell Bad Urban and Microcell Urban. The DOA estimation error produced by the Unitary-ESPRIT algorithm showed that this algorithm is more sensitive to the SNR changes than the other algorithms, in all the tested scenarios. Comparing the results when using the MDL algorithm to estimate the number of sources present and when using the estimate calculated by Unitary-ESPRIT it was seen that Unitary-ESPRIT produces more accurate estimates. Generally, the results showed the importance of power windows and good estimates for the number of sources in improving the accuracy of the DOA estimation algorithms.

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VI. References

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